Langevin Dynamics of Fluctuation Induced First Order Phase Transitions: self consistent Hartree Approximation

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The Langevin dynamics of a system exhibiting a Fluctuation Induced First Order Phase Transition is solved within the self consistent Hartree Approximation. Competition between interactions at short and long length scales gives rise to spatial modulations in the order parameter, like stripes in 2d and lamellae in 3d. We show that when the time scale of observation is small compared with the time needed to the formation of modulated structures, the dynamics is dominated by a standard ferromagnetic contribution plus a correction term. However, once these structures are formed, the long time dynamics is no longer pure ferromagnetic. After a quench from a disordered state to low temperatures the system develops growing domains of stripes (lamellae). Due to the character of the transition, the paramagnetic phase is metastable at all finite temperatures, and the correlation length diverges only at $T = 0$. Consequently, the temperature is a relevant variable, for $T > 0$ the system exhibits interrupted aging while for $T = 0$ the system ages for all time scales. Furthermore, for all T, the exponent associated with the aging phenomena is independent of the dimension of the system.

I. INTRODUCTION

Type-II superconductors [1], doped Mott insulators [2], ultrathin magnetic films[3, 4, 5], lipid monolayers [6], Raleigh-Benard convection [7], quantum Hall systems [8], are all systems that under appropriate conditions present stable phases characterized by the presence of modulated structures. The existence of these modulated structures is well understood on the basis of the Fluctuation Induced First Order Phase Transition Theory (FIFOT), first developed by Brazovskii [9]. This scenario predicts that systems in which the spectrum of fluctuations has a minimum in a shell in reciprocal space at a non zero wave vector, undergo a first order phase transition driven by fluctuations, in contrast to the second order transition predicted by mean field theory. Moreover, the strong degeneracy in the space of fluctuations induces the existence of many metastable structures at low temperatures, and since the experimentally observed structures are in general metastable, dynamical effects become very important. Unfortunately, the dynamical behavior of these systems is far from being understood.

The existence and stability of metastable structures and the nature of the nucleation processes in the context of the Brazovskii scenario where first studied by Hohenberg and Swift [7]. They obtained the free energy barriers to nucleation and the shape and size of critical droplets in the weak coupling limit. Gross et al. [10] compared the predictions of the self consistent Hartree approximation with direct simulations of the Langevin dynamics, confirming the validity of the approximation.

A classic example where the Brazovskii scenario has got strong support, both theoretically and experimentally, is in diblock copolymers [11, 12, 13]. These systems have interesting technological applications as self-assembling patterning media. Another well known example of this kind of system is the three dimensional Coulomb frustrated ferromagnet. Wolynes et al. [14] have shown, using a replica dynamical mean field theory, that below a characteristic temperature, an exponential number of metastable states appears in the system preventing long-range order. Furthermore, through Monte Carlo simulations [15] and also using a mode coupling analysis for the equilibrium Langevin dynamics of the Coulomb frustrated ferromagnet, Grousson et al. [16] have found an ergodicity breaking scenario in agreement with the predictions of Wolynes et al. These results resemble the behavior of many glass former systems, and two theoretical scenarios have been advocated in order to account for its phenomenology [17, 18]. The relevance of the mode coupling predictions to the dynamics of the system have nevertheless been questioned by Geissler et al. [19].

The experimental and theoretical study of thin film magnetic materials have led to similar questions [20, 21, 22]. Thin films and quasi-two dimensional magnetic materials have many important technological applications, for example in data storage and magnetic sensors [23]. In metallic uniaxial ferromagnetic films grown on a metallic substrate, like CoCu or FeCu, the system develops spontaneous stripe domains upon cooling below the Curie point. This is due to the competition between the exchange ferromagnetic short range interaction and the antiferromagnetic dipolar one, which is long range [4]. In the strong anisotropy or Ising limit, a self consistent Hartree approximation predicts the presence of a FIFOT for any value of the ratio between the ferro and antiferromagnetic coupling constants [21]. Nevertheless, the results from Monte Carlo simulations are far from conclusive,

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suggesting the presence of first order transitions only for a restricted range of the ratio of the coupling constants [24, 25]. The dynamics of these systems is also far from clear. Early work from Roland and Desai [26], who did simulations of the Langevin dynamics, concentrated on the early time regime, where modulation in the magnetization sets in. Later work with Monte Carlo simulations concentrated in some aspects of the out of equilibrium aging dynamics [27, 28] and the growing of stripe domains after a quench [29]. These works reveal a very rich phenomenology, with the appearance of complex phases reminiscent of liquid crystals, and strong metastability of the dynamics. All these facts point to the necessity of a more systematic study of the dynamical aspects of FIFOT.

In this work we solve the Langevin dynamics of a generic model undergoing a first order phase transition driven by fluctuations. To characterize the long time dynamics of the system, we study the fluctuation spectrum close to the wave vector k_0 representative of the modulated phases. We solve the dynamical equations in the Hartree approximation and show that, already within this approximation, the dynamics of the system is very rich and departs from the usual ferromagnetic case. A key observation is that, as we show in the next section, the spinodal of the high temperature disordered phase shifts to zero temperature in this approximation, and this has strong influence on the dynamics after a quench. In agreement with the equilibrium results, we show that the instability of the disordered phase appears only at $T = 0$, where the dynamics changes qualitatively. Nevertheless, the relaxation at finite temperature is far from trivial, showing the emergence of domains of stripes, which form a kind of mosaic state on top of the striped equilibrium phase.

The rest of the paper is organized as follows. In section II we present the model and show that, in the static self consistent approximation, it undergoes a FIFOT. In section III we introduce the Langevin dynamics. In section IV we present the general procedure to calculate the dynamical properties of the system in the Hartree approximation. Sections V and VI are the core of the paper, where the results on correlations and responses are presented and discussed. Some conclusions are presented in section VII. In two appendices we explain some technical details of the calculations.

II. A MODEL WITH A FLUCTUATION INDUCED FIRST ORDER TRANSITION

A classical model that undergoes a FIFOT may be defined by an attractive (ferromagnetic) short range interaction plus a competing, long-range repulsive (antiferromagnetic) interaction. In the simplest case of a scalar field, one can define an effective Landau-Ginzburg Hamiltonian of the form:

$$\mathcal{H}[\phi] = \int d^dx \left[\frac{1}{2}(\nabla\phi(\vec{x}))^2 + \frac{r}{2}\phi^2(\vec{x}) + \frac{u}{4}\phi^4(\vec{x})\right] + \frac{1}{2\delta}\int d^dx\, d^dx'\; \phi(\vec{x})\, J(\vec{x},\vec{x'})\, \phi(\vec{x}') \tag{1}$$

where $r < 0$ and $u > 0$. $J(\vec{x},\vec{x'}) = J(|\vec{x}-\vec{x'}|)$ represents a repulsive, isotropic, long range interaction and δ measures the relative intensity between the attractive and repulsive parts of the Hamiltonian. In the limit $\delta \to \infty$ one recovers the ferromagnetic $O(N)$ model (for $N = 1$) [30, 31, 32].

The $(u/4)\,\phi^4$ term introduces a non-linearity that makes an exact solution of the model an impossible task. To deal with this non-linearity, one must consider the introduction of some kind of perturbative analysis. The simplest resummation scheme is the self-consistent Hartree approximation, or large N limit. It consists of replacing one factor of ϕ^2 in the ϕ^4 term in the Hamiltonian by its average $\langle\phi^2\rangle$, to be determined self-consistently. There are six ways of choosing the two factors of ϕ to be paired in $\langle\phi^2\rangle$, so the Hamiltonian in the Hartree approximation takes the gaussian form:

$$\mathcal{H}[\phi] = \frac{1}{2}\int d^dx \left[(\nabla\phi(\vec{x}))^2 + r\,\phi^2(\vec{x}) + g\,\phi^2(\vec{x})\left\langle\phi^2(\vec{x})\right\rangle\right] + \frac{1}{2\delta}\int d^dx\, d^dx'\; \phi(\vec{x})\, J(|\vec{x}-\vec{x'}|)\, \phi(\vec{x'}) \tag{2}$$

where $g = 3u$. Introducing the Fourier transform:

$$\phi(\vec{x}) \;=\; \int \frac{d^dk}{(2\pi)^d}\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\phi}(\vec{k}) \tag{3}$$

$$\hat{\phi}(\vec{k}) \;=\; \int d^dx\, e^{-i\vec{k}\cdot\vec{x}}\, \phi(\vec{x}) \tag{4}$$

the Hamiltonian takes the form

$$\mathcal{H}[\phi] \;=\; \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\, A(k)\hat{\phi}(\vec{k})\hat{\phi}(-\vec{k}) + \frac{g}{2}\int \frac{d^dk_1}{(2\pi)^d}\int \frac{d^dk_2}{(2\pi)^d}\int \frac{d^dk_3}{(2\pi)^d}\, \hat{\phi}(\vec{k}_1)\hat{\phi}(\vec{k}_2)C(\vec{k}_3, -\vec{k}_1-\vec{k}_2-\vec{k}_3) \tag{5}$$

$$= \frac{1}{2}\int \frac{d^dk_1}{(2\pi)^d}\int \frac{d^dk_2}{(2\pi)^d}\, \hat{\phi}(\vec{k}_1)\left[A(k_1)\delta_{\vec{k}_1,-\vec{k}_2} + g\int \frac{d^dk}{(2\pi)^d}\, C(\vec{k}, -\vec{k}-\vec{k}_1-\vec{k}_2)\right]\hat{\phi}(\vec{k}_2) \tag{6}$$

$$= \frac{1}{2}\int \frac{d^dk_1}{(2\pi)^d}\int \frac{d^dk_2}{(2\pi)^d}\, \hat{\phi}(\vec{k}_1)\, A(\vec{k}_1,\vec{k}_2)\, \hat{\phi}(\vec{k}_2) \tag{7}$$

with

$$A(\vec{k}_1, \vec{k}_2) = A(k_1)\, \delta_{\vec{k}_1, -\vec{k}_2} + g \int \frac{d^d k}{(2\pi)^d}\, C(\vec{k}, -\vec{k} - \vec{k}_1 - \vec{k}_2) \tag{8}$$

In the previous expression $A(k) = r + k^2 + \hat{J}(k)/\delta$ and $C(\vec{k}, \vec{k}') = \langle \hat{\phi}(\vec{k}) \hat{\phi}(\vec{k}') \rangle$. Using that $A(\vec{k}_1, \vec{k}_2) = A(-\vec{k}_1, -\vec{k}_2)$ and that $C(\vec{k}, \vec{k}') = C_c(\vec{k}, \vec{k}') + m_{\vec{k}}\, m_{\vec{k}'}$, where $m_{\vec{k}} = \langle \hat{\phi}(\vec{k}) \rangle$ and $C_c(\vec{k}, \vec{k}')$ is the connected correlation function, we get finally the self-consistent Hartree equation for the connected correlator:

$$A(\vec{k}_1, \vec{k}_2) = \beta^{-1} C_c^{-1}(\vec{k}_1, \vec{k}_2) = A(k_1)\, \delta_{\vec{k}_1, -\vec{k}_2} + g \int \frac{d^d k}{(2\pi)^d} \left(m_{\vec{k}}\, m_{\vec{k}_1 + \vec{k}_2 - \vec{k}} + C_c(\vec{k}, \vec{k}_1 + \vec{k}_2 - \vec{k}) \right) \tag{9}$$

where $\beta = 1/k_B T$. In the paramagnetic phase, at high temperatures, all the order parameters $m_{\vec{k}} = 0$ and the correlation matrix is diagonal, i.e. $C_c(\vec{k}, \vec{k}') = S_c(\vec{k})\, \delta_{\vec{k}, -\vec{k}}$, with $S_c(\vec{k})$ the static structure factor. From Eq.(9) we have

$$\beta^{-1} S_c^{-1}(\vec{k}) = A(k) + g \int \frac{d^d k}{(2\pi)^d}\, S_c(\vec{k}) = r + k^2 + \frac{J(k)}{\delta} + g \int \frac{d^d k}{(2\pi)^d}\, S_c(\vec{k}) \tag{10}$$

Introducing the "renormalized mass":

$$\lambda = r + g \int \frac{d^d k}{(2\pi)^d}\, S_c(\vec{k}), \tag{11}$$

the structure factor becomes

$$S_c(\vec{k}) = \frac{T}{\lambda + k^2 + \frac{J(k)}{\delta}} \tag{12}$$

where we have set $k_B = 1$, and the renormalized mass has to be determined self-consistently from:

$$\lambda = r + gT \int \frac{d^d k}{(2\pi)^d}\, \frac{1}{\lambda + k^2 + \frac{J(k)}{\delta}} \tag{13}$$

An instability in this equation may appear when $\lambda = \lambda_c = -(k_0^2 + \frac{J(k_0)}{\delta})$, where k_0 is the wave vector which minimizes $A(k)$. Hence, the spinodal temperature T^* is determined by the equation

$$\beta^* (r^* - \lambda_c) = -g\, K_d \int_0^{\Lambda} \frac{k^{d-1}}{\lambda_c + k^2 + \frac{J(k)}{\delta}}\, dk \tag{14}$$

where K_d is the surface of a d-dimensional sphere. The integrand in the right hand side is always positive and has a singularity at $k = k_0$. Thus, the instability will be determined by the leading behavior of that integral, which can be estimated by expanding the denominator of the integrand around k_0:

$$\int_0^{\Lambda} \frac{k^{d-1}}{\lambda_c + k^2 + \frac{J(k)}{\delta}}\, dk \approx \int_{k_0 - \epsilon}^{k_0 + \epsilon} \frac{k^{d-1}}{c(k - k_0)^2} dk = \int_{-\epsilon}^{\epsilon} \frac{(k + k_0)^{d-1}}{k^2} dk = \infty$$

Therefore, the spinodal temperature always is depressed to zero, that is, $\beta^* r^* \rightarrow -\infty$. The fact that the isotropic phase is metastable at any finite temperature, a characteristic of the self-consistent nature of the fluctuations included in the model, will have important consequences on the dynamics after a quench at low temperatures. Nevertheless, it can be shown that below a melting temperature, the true equilibrium phase is a modulated one with characteristic wave vector k_0 and that a first order phase transition driven by fluctuations takes place [9].

III. LANGEVIN DYNAMICS

As usual, the Langevin dynamics for the scalar field $\phi(\vec{x}, t)$ is defined by:

$$\frac{\partial \phi(\vec{x}, t)}{\partial t} = -\frac{\delta \mathcal{H}[\phi]}{\delta \phi(\vec{x}, t)} + \eta(\vec{x}, t) \tag{15}$$

with, in addition, the following conditions for the thermal noise:

$$\begin{aligned} \langle \eta(\vec{x},t) \rangle &= 0 \\ \langle \eta(\vec{x},t)\eta(\vec{x'},t') \rangle &= 2T\delta(\vec{x}-\vec{x}')\delta(t-t') \end{aligned} \tag{16}$$

In this work, we consider uncorrelated initial conditions:

$$< \eta(\vec{x},0)\eta(\vec{x'},0) >= \Delta\, \delta(x-x') \tag{17}$$

If $\delta \to \infty$, the last term in equation (1) may be neglected and we keep only the short range part of the potential. At low temperatures this potential has two symmetric minima, $\phi(\vec{x}) = \pm\sqrt{-\frac{r}{u}}$, where $r < 0$. The dynamics of this model is well understood[30, 31], and is determined, below T_c, by the fixed point $T = 0$. At variance with the complete model, in the pure ferromagnetic case there is a continuous phase transition at a critical temperature T_c which, in the Hartree approximation, is different from the mean field critical temperature, with a lower critical dimension $d_l = 2$. In this case, the dynamics after a sub-critical quench corresponds to an usual domain growth, with a growth law $L(t) \propto t^{1/2}$, as in standard dynamical models with non-conserved order parameter.

IV. SELF CONSISTENT HARTREE APPROXIMATION

A. General Solution

In our case of interest the dynamical equation (15) reads:

$$\frac{\partial \phi(\vec{x},t)}{\partial t} = \nabla^2\phi(\vec{x},t) - r\phi(\vec{x},t) - u\,\phi^3(x,t) - \frac{1}{2\delta}\int d^dx'\, J(\vec{x},\vec{x}')\phi(\vec{x}',t) + \eta(\vec{x},t) \tag{18}$$

To extend consistently the known results for the equilibrium properties of the system to the study of the dynamics we keep the same resummation scheme. As before, in this approximation, the non-linear term ϕ^3 is substituted by $3\langle\phi^2(\vec{x},t)\rangle\phi(\vec{x},t)$ where the average is performed over the initial conditions and noise realizations. In such a way we obtain a linear equation in ϕ at the price of introducing a new parameter $< \phi^2 >$ to be determined self-consistently. To proceed, it is useful to go to Fourier space, in which we can write:

$$\frac{\partial \hat{\phi}(\vec{k},t)}{\partial t} = -[A(k) + I(t)]\,\hat{\phi}(\vec{k},t) + \hat{\eta}(\vec{k},t) \tag{19}$$

where

$$\begin{aligned} \langle \hat{\eta}(\vec{k},t) \rangle &= 0 \\ \langle \hat{\eta}(\vec{k},t)\hat{\eta}(\vec{k}',t') \rangle &= 2T\delta(\vec{k}+\vec{k}')\delta(t-t') \end{aligned} \tag{20}$$

$$I(t) = r + g\langle\phi^2(\vec{x},t)\rangle \tag{21}$$

$$A(\vec{k}) = k^2 + \frac{1}{\delta}\hat{J}(\vec{k},\vec{k}') \tag{22}$$

with initial conditions:

$$\begin{aligned} \langle \hat{\phi}_0(\vec{k}) \rangle &= 0 \\ \langle \hat{\phi}_0(\vec{k})\,\hat{\phi}_0(\vec{k}') \rangle &= (2\pi)^d\,\Delta\,\delta(\vec{k}+\vec{k}') \end{aligned} \tag{23}$$

From equation (19) it is easy to see that the general solution of the model may be written:

$$\hat{\phi}(\vec{k},t) = \hat{\phi}(\vec{k},0)\, R(\vec{k},t,0) + \int_0^t R(\vec{k},t,t')\, \hat{\eta}(\vec{k},t')\, dt' \tag{24}$$

where

$$R(\vec{k},t,t') = \frac{Y(t)}{Y(t')} e^{-A(\vec{k})(t-t')} \tag{25}$$

and $Y(t) = e^{\int_0^t dt' I(t')}$.

Our main task is now to find a solution for $Y(t)$, a function that encloses the unknown parameter introduced in the approximation. Following standard procedures [30, 31] it is easy to show that:

$$\frac{dK(t)}{dt} = 2rK(t) + 2g\Delta f(t) + 4gT \int_0^t dt' f(t-t')K(t') \tag{26}$$

where $K(t) = Y^2(t)$ and

$$f(t) = \int \frac{d^d k}{(2\pi)^d} e^{-2A(\vec{k})t}. \tag{27}$$

Equation (26) may be solve by Laplace transformation methods. If $\tilde{K}(p)$ and $\tilde{f}(p)$ are the Laplace transforms of $K(t)$ and $f(t)$ respectively, then equation (26) reduces to:

$$\tilde{K}(p) = \frac{2g\Delta \tilde{f}(p) + K(0)}{p - 2r - 4gT\tilde{f}(p)} \tag{28}$$

Technically, the problem has been reduced to the calculation of $\tilde{f}(p)$, to substitute it in equation (28) and to calculate the corresponding Bromwich integral for $K(t)$. Once $K(t)$ is known, all the dynamical quantities of the system may be easily calculated from integral relations.

V. CALCULATION OF $K(t)$

In this, rather technical section, we go through a series of approximations and assumptions, which allow us to compute the function $K(t)$ in the long time limit. We start the section presenting the approximation used to manage $A(\vec{k})$. It keeps the necessary ingredients to model a fluctuation induced first order phase transition. We then proceed to the calculation of $f(t)$ and finally $K(t)$, in the long time regime. We show that the cases $T = 0$ and $T > 0$ give rise to different physics, in agreement with the static calculations.

A. Approximation for $A(k)$

Unfortunately, the analytical calculation of $\tilde{f}(p)$ for general $A(\vec{k})$ is a hopeless task. We will simplify it, considering only cases in which $A(\vec{k})$ depends on the modulus of $\vec{k}$, $A(\vec{k}) = A(k)$ (isotropic interactions). Since we are interested in the long time dynamics of the model, and we know that the equilibrium phases are characterized by the existence of a non-trivial wave vector $k_0 \neq 0$, at which the spectrum of fluctuations has a maximum, it is then natural to develop $A(k)$ close to k_0:

$$A(k) = A_0 + \frac{A_2}{2}\,(k-k_0)^2 + \mathcal{O}[(k-k_0)^3] \tag{29}$$

where

$$\begin{aligned} A_0 &= A(k_0) \\ A_2 &= \left.\frac{d^2 A}{dk^2}\right|_{k=k_0} \qquad (30) \\ & etc \qquad (31) \end{aligned}$$

with $A_2 > 0$.

Note that, if t is large enough, this approximation is valid not only for models with long range interactions, but in general is a good starting point to study other systems whose spectrum of fluctuations has an isotropic minimum at a non-zero wave vector. Therefore, the reader must keep in mind that the results of the next sections are valid in a context more general than the one represented by the Hamiltonian (1).

From a technical point of view, one may note that A_0 is irrelevant to the dynamical behavior of the system. In fact, from equation (19), one can easily see that it is equivalent to a rescaling of r, and therefore to a shift in the critical temperature of the system. Therefore, from now on it will be neglected in our calculations.

B. Results for $\tilde{f}(p)$

With the previous assumptions for $A(k)$, we may write $f(t)$ as:

$$f(t) = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int \frac{d^d k}{(2\pi)^d} e^{-A_2 t (k-k_0)^2} \tag{32}$$

whose Laplace transform becomes

$$\tilde{f}(p) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{p + A_2 (k-k_0)^2} \tag{33}$$

Next we analyze the behavior of this integral when $p \simeq 0$. Adding a cutoff factor in the integrals in order to regularize the behavior for large wave vectors:

$$\tilde{f}(p) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{-(k-k_0)^2/\Lambda^2}}{p + A_2(k-k_0)^2} = \frac{2\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \int_0^\infty dk \, \frac{k^{d-1} e^{-(k-k_0)^2/\Lambda^2}}{p + A_2(k-k_0)^2} \tag{34}$$

Then, after simple algebra equation (34) becomes:

$$\tilde{f}(p) = \frac{2\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \frac{1}{(pA_2)^{1/2}} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk \, e^{-pk^2/A_2\Lambda^2}}{1+k^2} \left[k_0 + \left(\frac{p}{A_2}\right)^{1/2} k \right]^{d-1} . \tag{35}$$

Expanding the binomial inside the integral we obtain the following expression for general dimensions:

$$\tilde{f}(p) = \frac{2\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \frac{1}{A_2} \left(\frac{A_2}{p}\right)^{1/2} \sum_{j=0}^{d-1} \binom{d-1}{j} \left(\frac{p}{A_2}\right)^{j/2} k_0^{d-1-j} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk \, e^{-pk^2/A_2\Lambda^2}}{1+k^2} k^j \tag{36}$$

with $d \in \mathcal{Z}$.

At this point two limit cases are possible and will be treated separately below. If $(\frac{A_2}{p})^{1/2}|k_0| \to 0$ the time scale of observation is such that the stripes are not completely formed, $\frac{1}{p} < \frac{1}{k_0^2}\frac{1}{A_2}$. In this limit we recover the dynamic properties of the pure ferromagnet for $k_0 \sim 0$. On the other hand, if $(\frac{A_2}{p})^{1/2}|k_0| \to \infty$ the stripes are already formed and the interaction among them will be responsible of the dynamical properties of the system. Once this, more interesting limit, is taken, it is impossible to recover the pure ferromagnetic behavior.

1. *Stripes in formation*

Defining:

$$F_j(k_0, p) = \left(\frac{p}{A_2}\right)^{j/2} k_0^{d-1-j} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk \, e^{-pk^2/A_2\Lambda^2}}{1+k^2} k^j \tag{37}$$

and writing it as a Taylor series expansion, for $k_0 \sim 0$: $F_j(k_0,p) = F_j(0,p) + F_j'(0,p)k_0 + O(k_0^2)$, we get:

$$F_j(0,p) = \begin{cases} (\frac{p}{A_2})^{(d-1)/2} \int_0^\infty dk \frac{k^{d-1}}{1+k^2} e^{-\frac{p}{A_2}\frac{k^2}{\Lambda^2}} & \text{if } j = d-1 \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

and

$$F_j'(0,p) = \begin{cases} (\frac{p}{A_2})^{(d-2)/2} \int_0^\infty dk \frac{k^{d-1}}{1+k^2} e^{-\frac{p}{A_2}\frac{k^2}{\Lambda^2}} & \text{if j=d-2} \\ 0 & \text{otherwise} \end{cases} \tag{39}$$

Therefore, up to first order in k_0, $F(k_0,p)$ becomes:

$$F_j(k_0,p) = (\frac{p}{A_2})^{(d-1)/2} \int_0^\infty dk \frac{k^{d-1}}{1+k^2} e^{-\frac{p}{A_2}\frac{k^2}{\Lambda^2}} + k_0 (\frac{p}{A_2})^{(d-1)/2} \int_0^\infty dk \frac{k^{d-1}}{1+k^2} e^{-\frac{p}{A_2}\frac{k^2}{\Lambda^2}}, \tag{40}$$

and $\tilde{f}(p)$ may be written as:

$$\tilde{f}(p) = p^{\frac{d}{2}-1} \left[a + b (\frac{A_2}{p})^{1/2} k_0 \right] \tag{41}$$

with $a = \frac{1}{(4\pi)^{\frac{d}{2}} A_2}$ and $b = -\frac{1}{(4\pi)^{d/2} A_2^{3/2}} \frac{\pi \sec(\frac{\pi d}{2})}{\Gamma(\frac{d}{2})}$.

From equation (41) it comes out that the presence of modulated phases appear in the dynamics as a correction in $\tilde{f}(p)$ to the usual ferromagnetic case [30]. One must remember, however, that this is true provided the second term within the brackets is small, i.e. during the formation of the modulated structures.

2. *Stripes formed*

On the other hand, for the limit $(\frac{A_2}{p})^{1/2}|k_0| \to \infty$, stripes of sizes $1/|k_0|$ are already formed, and the dynamical properties of the system are defined by their interactions. The Taylor expansion, for k_0 finite and $p \to 0$, is written as: $F_j(k_0,p) = F_j(k_0,0) + F_j'(k_0,0)p + O(p^2)$. Then,

$$F_j(k_0,0) = \begin{cases} k_0^{d-1} \int_{-\infty}^\infty \frac{dk}{1+k^2} = \pi k_0^{d-1} & \text{if } j = 0 \\ 0 & \text{otherwise} \end{cases} \tag{42}$$

and developing as before the first order derivative with respect to p, and taking the limit $p \to 0$, $\tilde{f}(p)$ becomes:

$$\tilde{f}(p) = \frac{2\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \frac{1}{A_2} (\frac{A_2}{p})^{1/2} \left(\pi k_0^{d-1} + A_2 k_0^{d-3} \int_{-\infty}^\infty dk \frac{k^2}{1+k^2} e^{-\frac{p}{A_2}\frac{k^2}{\Lambda^2}} - \frac{k_0^{d-2}}{2} \sqrt{\frac{p}{A_2}} \right) \tag{43}$$

This last expression shows that for small p, $\tilde{f}(p) = a + bp^{-1/2}$ with $a < 0$, independently of the system dimension. This limit was also explicitly calculated for $d = 1, 2$ and 3, confirming the series analysis. The calculations are shown in Appendix A.

Summarizing this subsection, $\tilde{f}(p)$ was calculated in two limiting cases. In the first case, the system is still evolving and the stripes are not formed. The dynamical properties resemble the ones of the pure ferromagnet plus a correction term. Once the stripes are formed, the dynamic changes qualitatively, and one gets that $\tilde{f}(p) = a + bp^{-1/2}$, independently of the dimensionality. From now on, we will use this expression in future calculations, and only when necessary we will give explicit values for a and b.

C. The function $K(t)$

By definition:

$$K(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dp\, e^{pt}\, \tilde{K}(p). \tag{44}$$

with $\tilde{K}(p)$ defined by (28), where the regularization factors in $\tilde{f}(p)$ can be disregarded in the long time limit. Then:

$$\tilde{K}(p) = \frac{2g\Delta a + 2g\Delta b p^{-1/2} + 1}{p - 2A_0 - 2r - 4gTa - 4gTbp^{-1/2}} \tag{45}$$

Because $\tilde{f}(p) = a + bp^{-1/2}$, $\tilde{K}(p)$ has a branch point at $p = 0$, and the denominator varies in the domain $(-\infty, \infty)$.

Simplifying the notation we can write

$$\tilde{K}(p) = \frac{A + Bp^{-1/2}}{p - C - Dp^{-1/2}} = \frac{Ap^{1/2} + B}{p^{3/2} - Cp^{1/2} - D}. \tag{46}$$

with $A = 1 + 2g\Delta a$, $B = 2g\Delta b$, $C = 2r + 4gTa$ and $D = 4gTb$.

The denominator of equation (46) has three poles. Through a careful analysis it is possible to show that one pole is real and positive for all temperatures. The other two are complex conjugate with negative real part. We have to solve:

$$\begin{aligned} K(t) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dp\, e^{pt}\, \frac{Ap^{1/2} + B}{p^{3/2} - Cp^{1/2} - D} \\ &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dp\, e^{pt}\, \frac{Ap^{1/2} + B}{(p^{1/2} - x)(p^{1/2} - z)(p^{1/2} - z^*)}, \end{aligned} \tag{47}$$

where $x^2 \in \mathcal{R}$, $z^2 \in \mathcal{C}$ and $(z^2)^*$ is the complex conjugate of z^2. After a lengthy computation (see Appendix B), we get:

$$\begin{aligned} K(t) &= \frac{2\,x\,(A\,x + B)}{(x - z)(x - z^*)}\, e^{x^2 t} + \frac{2\,z\,(A\,z + B)}{(z - x)(z - z^*)}\, e^{z^2 t} + \frac{2\,z^*\,(A\,z^* + B)}{(z^* - x)(z^* - z)}\, e^{(z^2)^* t} \\ &- \frac{1}{\pi} \int_0^{\infty} dr\, e^{-rt}\, \frac{B\,r^{3/2} + (BC - AD)r^{1/2}}{D^2 + (r^{3/2} + C\,r^{1/2})^2} \end{aligned} \tag{48}$$

Now, one must distinguish carefully the cases $T > 0$ and $T = 0$. Note, for example, that the real parts of the complex poles are negative even for $T \to 0$, while the real pole is always positive going to zero at $T = 0$, where the physics changes qualitatively.

1. $T > 0$

In this case one may neglect the contributions from the complex conjugate poles, since their real parts have decaying exponential functions. On the other hand, the last integral in equation (48) may be easily estimated noting that: $(r^{3/2} + C\,r^{1/2})^2 = r^3 + 2\,C\,r^2 + C^2\,r$, and that for long times $t \to \infty$ the dominant contributions will come from $r \ll 1$. We end with:

$$\frac{BC - AD}{D^2} \int_0^{\infty} dr\, e^{-rt}\, r^{1/2} = \frac{BC - AD}{D^2} \frac{\sqrt{\pi}}{2\,t^{3/2}}. \tag{49}$$

Therefore,

$$K(t) \approx \frac{2\,x\,(A\,x + B)}{(x - z)(x - z^*)}\, e^{x^2 t} - \frac{BC - AD}{2\sqrt{\pi} D^2} \frac{1}{t^{3/2}}. \tag{50}$$

where the last term goes to zero for $t \to \infty$.

2. $T = 0$

In this case one must note that $x(T) \to T$, therefore the contribution from the real pole dissapears. At the same time, the complex poles converge to a single real pole that gives rise to a decaying exponential function.

Moreover, the expansion used to calculate the last integral in (48) is not longer valid. Being $D = 0$ one finds that, for large t:

$$B \int_0^\infty dr \frac{e^{-rt}}{r^{\frac{3}{2}} + Cr^{\frac{1}{2}}} \sim \frac{B}{\sqrt{\pi} C t^{\frac{1}{2}}} \tag{51}$$

and

$$K(t) = Ae^{Ct} - \frac{B}{C\pi} \frac{\Gamma(1/2)}{\sqrt{t}} \tag{52}$$

with $C < 0$. The first term comes from the limit as $T \to 0$ of the two complex poles and is obviously subdominant in this analysis.

Already with these results at hand one must note two important differences with the usual ferromagnetic coarsening. The first one is that here the temperature is a relevant variable. While for $T > 0$ the relaxation will be dominated by exponential (paramagnetic) contributions, for $T = 0$ the relaxation will be power-like. The second one is that, excluding irrelevant prefactors, the long time dynamics, for all temperatures, *is independent of the system dimensionality*.

VI. RESPONSE AND CORRELATION FUNCTIONS

In this section we present the main physical results of the paper, regarding the behavior of correlation and response functions.

As seen in Section (IV), the two-times response function in Fourier space is given by:

$$R(k, t, t') = \frac{Y(t')}{Y(t)} e^{-A(k)(t-t')} \tag{53}$$

with $Y(t) = \sqrt{K(t)}$. Defining the two-times structure factor:

$$\langle \phi(\vec{k}, t)\, \phi(\vec{k}', t') \rangle = (2\pi)^d\, \delta(\vec{k} + \vec{k}')\, C(\vec{k}, t, t') \tag{54}$$

and using (20), (21) and (24), one can show that:

$$C(\vec{k}, t, t') = \Delta\, R(\vec{k}, t, 0)\, R(\vec{k}, t', 0) + 2T \int_0^{t'} R(\vec{k}, t, s)\, R(\vec{k}, t', s)\, ds \tag{55}$$

As the physics at finite temperature is different from that at $T = 0$, we will analyze both cases separately. Also, note that, for $T \neq 0$, the leading contribution to $K(t)$ is exponential, and consequently, to leading order, correlations and responses will be stationary, consistent with a paramagnetic phase. Nevertheless, the algebraic subdominant contribution precludes the presence of a transient non-stationary dynamics, with time scales that can be large for low enough temperatures. Consequently, we will keep the subleading contribution also and analyze its effect on the dynamics, showing that it leads to interrupted aging in correlations and responses.

A. $T > 0$

From equation (50) we find:

$$Y(t) = \sqrt{\frac{2\,x\,(A\,x + B)}{(x - z)(x - z^*)}\, e^{x^2 t} - \frac{BC - AD}{2\sqrt{\pi} D^2} \frac{1}{t^{3/2}}} \tag{56}$$

and defining $C_2 = \frac{2\,x\,(A\,x+B)}{(x-z)(x-z^*)}$ and $C_1 = -\frac{BC-AD}{2\sqrt{\pi}D^2}$ to simplify the notation, one gets for large t:

$$Y(t) = C_1^{\frac{1}{2}} e^{\frac{1}{2}x^2 t} (1 + \frac{C_2}{2C_1} \frac{e^{-x^2 t}}{t^{\frac{3}{2}}}) \tag{57}$$

Then, the response function for $T > 0$ is:

$$R(k,t,t') = e^{-\frac{1}{2}[x^2+A_2(k-k_0)^2](t-t')} \left[1 + \frac{C_2}{2C_1}\frac{e^{-x^2t'}}{t'^{3/2}}\right] \tag{58}$$

The two-time correlation function is given by equation (55). Then, using (58) and defining $B(k) = x^2 + A_2(k-k_0)^2$ we get:

$$C(k,t,t') = \Delta\, e^{-\frac{1}{2}B(k)(t+t')} + 2T\, e^{-\frac{1}{2}B(k)(t+t')} \int_0^{t'} ds\, e^{-\frac{1}{2}B(k)s} \left(1 + \frac{C_2}{2C_1}\frac{e^{-x^2s}}{s^{3/2}}\right)^2 \tag{59}$$

Performing the integration in (59) and setting $\tau = t - t'$ one gets:

$$C(k,\tau,t') = e^{-\frac{1}{2}B(k)\tau} \left[\frac{2T}{B(k)} + \left(\Delta - \frac{2T}{B(k)}\right) e^{-B(k)t'} - \frac{C_2}{C_1}\frac{e^{-\frac{1}{2}x^2t'}}{\sqrt{t'}}\right] \tag{60}$$

The second term within parenthesis goes rapidly to zero, while the $\sqrt{t'}$ in the third term reflects the presence of interrupted aging in the system. On the other hand, keeping the stationary part and for $t = t'$, we obtain the static structure factor:

$$C(\vec{k}) = \lim_{t\to\infty} C(\vec{k},t) = \frac{2T}{B(k)} = \frac{2T}{x^2 + A_2(k-k_0)^2} \tag{61}$$

which, as expected, shows a characteristic peak at $k = k_0$.

The correlation function in real space is given by:

$$C(\vec{x}) = \int_{-\infty}^{\infty} \frac{d^dk}{(2\pi)^d}\, C(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} \tag{62}$$

In d dimensions:

$$C(r) = \frac{1}{(2\pi)^{d/2}} \int_0^{\infty} \left(\frac{1}{kr}\right)^{\frac{d-2}{2}} J_{\frac{d-2}{2}}(kr)\, \frac{k^{d-1}\,dk}{x^2 + A_2(k-k_0)^2}, \tag{63}$$

where $J_\nu(x)$ is a Bessel function of the first kind. In the limit $kr \to \infty$,

$$C(r) \approx \frac{1}{(2\pi)^{d/2}} \int_0^{\infty} \left(\frac{1}{kr}\right)^{\frac{d-2}{2}} \sqrt{\frac{2}{\pi kr}} \cos\left(kr - \frac{(d-1)\pi}{4}\right) \frac{k^{d-1}\,dk}{x^2 + A_2(k-k_0)^2} \tag{64}$$

This integral can be solved using the theorem of residues in the complex plane. The final result is:

$$C(r) \propto \cos\left(k_0 r - \psi\right) \frac{e^{-r/\xi}}{r^{\frac{d-1}{2}}}. \tag{65}$$

where $\psi = \frac{(d-1)\pi}{4} - \tan^{-1}\left(\frac{(d-1)x}{2k_0\sqrt{A_2}}\right)$. We see the presence of a correlation length and a modulation length. The correlation length is given by

$$\xi(T) = \sqrt{\frac{A_2}{x^2}} \tag{66}$$

As mentioned before, at low temperatures, $x(T) \propto T$, and consequently the correlation length diverges at $T = 0$ as $\xi(T) \propto 1/T$. Then, one can conclude that after a quench from the disordered phase to a very low temperature the system breaks into regions inside which there is modulated order, or stripes. No long range order is observed. A transition is approached at $T = 0$, where the correlation length diverges and stripe order sets in. In this respect, it is interesting to see also what happens with the correlation function at $T = 0$.

B. $T = 0$

For $T = 0$ we may neglect the decaying exponential and simple algebra gives:

$$R(k,t,t') = (\frac{t}{t'})^{\frac{1}{4}} e^{-\frac{1}{2}[x^2 + A_2(k-k_0)^2](t-t')} \tag{67}$$

Substituting (67) in (55) it is easy to prove that

$$C(k,t,t') = \Delta R(k,t,0) R(k,t',0) = \Delta(tt')^{\frac{1}{4}} e^{-\frac{A_2}{2}(k-k_0)^2(t+t')} \tag{68}$$

and making $t = t'$ we obtain

$$C(k,t) = \frac{\Delta}{W} t^{\frac{1}{2}} e^{-A_2(k-k_0)^2(t)} \tag{69}$$

with $W = \frac{B\Gamma(\frac{1}{2})}{\pi|C|}$.

For $d = 1$ the spatial correlation function has the form:

$$C(r,t) = \frac{\Delta\sqrt{\pi}}{2W} \cos{(k_0 r)}\; e^{-\frac{r^2}{4t}} erf(k_0\sqrt{t}), \tag{70}$$

where $erf(x)$ is the error function. This result is consistent, for large t, with the appearance of modulated structures and long-range order. Nevertheless note that, from Eq.(65), a critical decay of correlations is observed for $d > 1$.

The stability of the paramagnetic phase until $T = 0$ is obtained also in a purely static calculation of the phase diagram in the self consistent (Hartree) approximation. Nevertheless, the same calculation shows that the disordered phase is only metastable, the stripe phase has a lower free energy below a finite critical temperature and is thus the true thermodynamic equilibrium of the system. Our calculations show that the Langevin Dynamics within the Hartree approximation reproduces this scenario. A quench from a disordered phase to $T > 0$ gives rise to a paramagnetic-like dynamics reflecting the metastability of this phase. Only at $T = 0$, the spinodal is reached, and the system shows a coarsening non-equilibrium dynamics, with diverging time scales in the thermodynamic limit.

VII. CONCLUSIONS

In this work we have calculated within the Hartree approximation the exact long time dynamics of a model system exhibiting a Fluctuation Induced First Order Phase Transition. We motivate our work starting with a Hamiltonian with short range ferromagnetic interaction and long range antiferromagnetic interactions, but our results are valid in general for the long time dynamics of any system exhibiting Fluctuation Induced First Order Phase Transitions, provided that the spectrum of the fluctuations is isotropic. We present explicit expressions for one and two time correlation and response functions and show that the dynamics converges to known static results in the Hartree approximation. Our results show that the dynamics of the system may be decomposed in two stages, first the modulated phases form, and during this stage the dynamics follows the usual coarsening scenario for a ferromagnetic system: it is dominated by the zero temperature fixed point and depends on the dimensionality of the system. Once these modulated structures are formed, the dynamics changes qualitatively. It becomes independent of the system dimension and the temperature becomes a relevant variable. For $T > 0$ the system exhibits interrupted aging and a standard paramagnetic relaxation for large times, dominated by the presence of metastable states. At low temperatures domanins of stripes are formed. At $T = 0$ the correlation length diverges and stripe order sets in. The system ages for all the time scales following a coarsening dynamics that searches the equilibrium state. Moreover, the exponent associated with this aging dynamics is independent of the system dimensionality.

In this work we have explored only the presence of positional order, through the calculation of the correlations of the field $\phi(\vec{x},t)$. It would be interesting to compute also orientational observables, which are known to be relevant for these kind of systems, and give rise to nematic-like order [25]. Other interesting questions that can be addressed starting from the present calculations are the possible nucleation of stripe phases in the paramagnetic state, which eventually should lead to the first order transition predicted within the static Hartree approximation. Also the possible presence of freezing in the low temperature dynamics could be addressed withing a refined approximation, like mode-coupling or the self-consistent screening approximation.

APPENDIX A

Here we show the explicit calculation of $\tilde{f}(p)$ in the limit $p \to 0$ for $d = 1$, $d = 2$ and $d = 3$. We begin from Eq.(36) in each case, and develop the sums and integrals.

d=1

$$\tilde{f}(p) = \frac{1}{\pi (pA_2)^{1/2}} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} \tag{A1}$$

Now,

$$\begin{aligned} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} &= \int_{-\infty}^{\infty} \frac{dk}{1+k^2} - \int_{-\infty}^{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} \\ &= \pi - \int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} \end{aligned} \tag{A2}$$

The last equality is due to the integrand be an even function. For $p \to 0$ the last integral gives:

$$\int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} = \left(\frac{p}{A_2}\right)^{1/2} \frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|} + \mathcal{O}(p^{3/2}) \tag{A3}$$

Then,

$$\begin{aligned} \tilde{f}(p) &= \frac{1}{\pi (pA_2)^{1/2}} \left\{ \pi - \left(\frac{p}{A_2}\right)^{1/2} \frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|} + \mathcal{O}(p^{3/2}) \right\} \\ &= \frac{1}{A_2^{1/2}} p^{-1/2} - \frac{e^{-|k_0|^2/\Lambda^2}}{\pi A_2 |k_0|} + \mathcal{O}(p) \end{aligned} \tag{A4}$$

d=2

$$\tilde{f}(p) = \frac{k_0}{2\pi (pA_2)^{1/2}} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} \left[1 + \left(\frac{p}{A_2}\right)^{1/2} \frac{k}{k_0} \right] \tag{A5}$$

Here we have to solve two integrals. The first one was already solved for the case $d = 1$:

$$\int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2} = \pi - \left(\frac{p}{A_2}\right)^{1/2} \frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|} + \mathcal{O}(p^{3/2}) \tag{A6}$$

The second integral is:

$$\int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk\ k}{1+k^2} e^{-pk^2/A_2\Lambda^2} = \int_{-\infty}^{\infty} - \int_{-\infty}^{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|} = \int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk\ k}{1+k^2} e^{-pk^2/A_2\Lambda^2} \tag{A7}$$

because the integrand is an odd function. The last integral can be approximated as:

$$\int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk\ k}{1+k^2} e^{-pk^2/A_2\Lambda^2} = \int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{k} e^{-pk^2/A_2\Lambda^2} \left[1 - \frac{1}{k^2} + \mathcal{O}(k^{-4}) \right] \tag{A8}$$

Now

$$\int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{k} e^{-pk^2/A_2\Lambda^2} = \frac{1}{2}\Gamma(0, \frac{|k_0|^2}{\Lambda^2}) \\ = -\frac{\gamma}{2} - \frac{1}{2}\,\log\left(\frac{|k_0|^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{|k_0|^2}{\Lambda^2}\right), \quad \text{(A9)}$$

where γ is the Euler constant. The other integral

$$\int_{\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{k^3} e^{-pk^2/A_2\Lambda^2} = \frac{1}{2}\left\{\frac{p\,e^{-|k_0|^2/\Lambda^2}}{A_2|k_0|^2} - \frac{p\,\Gamma(0, \frac{|k_0|^2}{\Lambda^2})}{A_2\Lambda^2}\right\} \\ = \frac{p}{2A_2}\left\{\frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|^2} + \frac{\gamma}{\Lambda^2} + \frac{1}{\Lambda^2}\log\left(\frac{|k_0|^2}{\Lambda^2}\right)\right\} \quad \text{(A10)}$$

Then,

$$\int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk\;k}{1+k^2} e^{-pk^2/A_2\Lambda^2} = -\frac{\gamma}{2} - \frac{1}{2}\log\left(\frac{|k_0|^2}{\Lambda^2}\right) - \frac{p}{2A_2}\frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|^2} + \mathcal{O}(\Lambda^{-2}) \quad \text{(A11)}$$

Finally,

$$\tilde{f}(p) = \frac{k_0}{2\pi A_2^{1/2}}\left\{\pi p^{-1/2} - \frac{1}{A_2^{1/2}}\frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|} + \frac{1}{k_0 A_2^{1/2}}\left[-\frac{\gamma}{2} - \frac{1}{2}\log\left(\frac{|k_0|^2}{\Lambda^2}\right) + \mathcal{O}(p)\right]\right\} \\ = \frac{k_0}{2A_2^{1/2}}p^{-1/2} - \frac{e^{-|k_0|^2/\Lambda^2}}{2\pi A_2} - \frac{\gamma}{4\pi A_2} - \mathcal{O}(\log\Lambda^{-2}) \quad \text{(A12)}$$

d=3

In this case

$$\tilde{f}(p) = \frac{2\pi^{3/2}k_0^2}{(2\pi)^3\Gamma(3/2)(pA_2)^{1/2}} \int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk}{1+k^2} e^{-pk^2/A_2\Lambda^2}\left[1 + \left(\frac{p}{A_2}\right)^{1/2}\frac{k}{k_0} + \left(\frac{p}{A_2}\right)\frac{k^2}{k_0^2}\right] \\ = \frac{2\pi^{3/2}k_0^2}{(2\pi)^3\Gamma(3/2)(pA_2)^{1/2}}\left\{\pi - \left(\frac{p}{A_2}\right)^{1/2}\frac{e^{-|k_0|^2/\Lambda^2}}{|k_0|} + \frac{1}{k_0}\left(\frac{p}{A_2}\right)^{1/2}\left[-\frac{\gamma}{2} - \frac{1}{2}\,\log\left(\frac{|k_0|^2}{\Lambda^2}\right)\right] + \right. \\ \left. \frac{1}{k_0^2}\left(\frac{p}{A_2}\right)\int_{-\left(\frac{A_2}{p}\right)^{1/2}|k_0|}^{\infty} \frac{dk\;k^2}{1+k^2} e^{-pk^2/A_2\Lambda^2}\right\} \quad \text{(A13)}$$

Again, the contribution of the dominant term is of order $p^{-1/2}$, proving that, in agreement with the series expansion in the text, the behavior of $\tilde{f}(p)$ is independent of dimensionality.

APPENDIX B

For $\tilde{f}(p) == a + b\,p^{-1/2}$ we have

$$K(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dp\; e^{pt}\;\frac{Ap^{1/2}+B}{p^{3/2} - Cp^{1/2} - D} \\ = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dp\; e^{pt}\;\frac{Ap^{1/2}+B}{(p^{1/2}-x)(p^{1/2}-z)(p^{1/2}-z^*)}, \quad \text{(B1)}$$

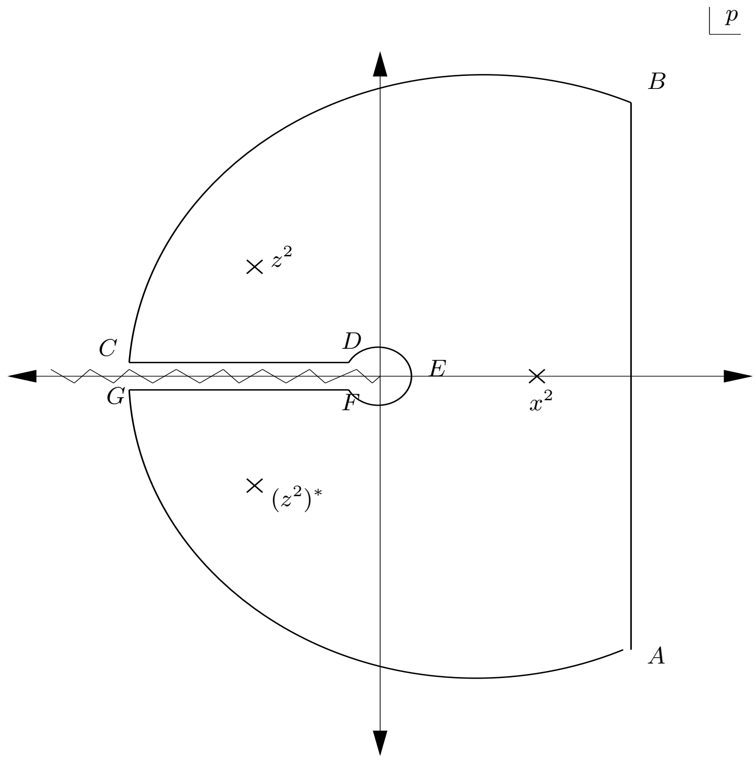

FIG. 1: Path of integration for the inverse Laplace transform

where $x^2 \in \mathcal{R}$, $z^2 \in \mathcal{C}$ and $(z^*)^2$ is the complex conjugate of z^2. The three poles x^2, z^2 and $(z^*)^2$ are simple and the residues are:

$$\begin{aligned} \lim_{p\to x^2}(p-x^2)\,e^{pt}\,\tilde{K}(p) &= \frac{(Ax+B)\,e^{x^2t}\,2x}{(x-z)(x-z^*)} \\ \lim_{p\to z^2}(p-z^2)\,e^{pt}\,\tilde{K}(p) &= \frac{(Az+B)\,e^{z^2t}\,2z}{(z-x)(z-z^*)} \\ \lim_{p\to (z^*)^2}(p-(z^*)^2)\,e^{pt}\,\tilde{K}(p) &= \frac{(Az^*+B)\,e^{(z^*)^2t}\,2z^*}{(z^*-x)(z^*-z)} \end{aligned} \tag{B2}$$

In order to perform the integral, consider the contour in Fig.(1). We choose the branch cut to be the negative real axis $(-\infty,0]$. From (B1) we conclude that the paths BC, DEF and GA do not contribute. Then:

$$\int_A^B + \int_C^D + \int_F^G = 2\pi i \sum Res \tag{B3}$$

Define $p = r\,e^{i\theta}$,

$$\begin{aligned} \int_C^D dpe^{pt}\frac{Ap^{1/2}+B}{p^{3/2}-Cp^{1/2}-D} &= \int_\infty^0 d(r\,e^{-i\pi})\,e^{r\,e^{i\pi}t}\frac{A\,r^{1/2}\,e^{i\pi/2}+B}{r^{3/2}\,e^{3i\pi/2}-C\,r^{1/2}\,e^{i\pi/2}-D} \\ &= -\int_\infty^0 dr\;e^{-rt}\frac{B+iA\,r^{1/2}}{-i\,r^{3/2}-i\,C\,r^{1/2}-D} \end{aligned} \tag{B4}$$

$$\begin{aligned} \int_F^G dpe^{pt}\frac{Ap^{1/2}+B}{p^{3/2}-Cp^{1/2}-D} &= \int_F^G d(r\,e^{-i\pi})\,e^{r\,e^{-i\pi}t}\frac{A\,r^{1/2}\,e^{-i\pi/2}+B}{r^{3/2}\,e^{-3i\pi/2}-C\,r^{1/2}\,e^{-i\pi/2}-D} \\ &= -\int_0^\infty dr\;e^{-rt}\frac{B-iA\,r^{1/2}}{i\,r^{3/2}+i\,C\,r^{1/2}-D} \end{aligned} \tag{B5}$$

Then,

$$\begin{aligned}
\int_C^D + \int_F^G &= \int_0^\infty dr\, e^{-rt} \left\{ \frac{-B + iA\,r^{1/2}}{i(r^{3/2} + C\,r^{1/2}) - D} - \frac{B + iA\,r^{1/2}}{i(r^{3/2} + C\,r^{1/2}) + D} \right\} \\
&= \int_0^\infty dr\, e^{-rt} \left\{ \frac{B - iA\,r^{1/2}}{D - i(r^{3/2} + C\,r^{1/2})} - \frac{B + iA\,r^{1/2}}{D + i(r^{3/2} + C\,r^{1/2})} \right\} \\
&= \int_0^\infty dr\, e^{-rt} \frac{[D + i(r^{3/2} + C\,r^{1/2})](B - iA\,r^{1/2}) - [D - i(r^{3/2} + C\,r^{1/2})](B + iA\,r^{1/2})}{D^2 + (r^{3/2} + C\,r^{1/2})^2} \qquad \text{(B6)}
\end{aligned}$$

The numerator has the form:

$$\begin{aligned}
xy^* - x^*y &= (\Re x + i\Im x)(\Re y - i\Im y) - (\Re x - i\Im x)(\Re y + i\Im y) \\
&= \Re x \Re y - i\Re x \Im y + i\Re y \Im x + \Im x \Im y - i\Re x \Im y + i\Re y \Im x - \Re x \Re y - \Im x \Im y \\
&= -2i\Re x \Im y + 2i\Re y \Im x \qquad \text{(B7)}
\end{aligned}$$

Then,

$$\begin{aligned}
\int_D^C + \int_F^G &= 2i \int_0^\infty dr\, e^{-rt} \frac{[B(r^{3/2} + C\,r^{1/2}) - A\,D\,r^{1/2}]}{D^2 + (r^{3/2} + C\,r^{1/2})^2} \\
&= 2i \int_0^\infty dr\, e^{-rt} \frac{B\,r^{3/2} + (BC - AD)r^{1/2}}{D^2 + (r^{3/2} + C\,r^{1/2})^2}, \qquad \text{(B8)}
\end{aligned}$$

and

$$K(t) = \sum Res - \frac{1}{2\pi i} \left\{ \int_D^C + \int_F^G \right\} \qquad \text{(B9)}$$

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